Noncontact heat transfer between two metamaterials

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Dispersive metamaterials are artificial composite media with magnetodielectric losses so that they naturally radiate in their surrounding. Here we theoretically investigate both radiative and nonradiative heat exchanges between two metamataterials. We show that the presence of magnetic surface modes and the ferromagnetic behavior of materials close to their magnetic resonance yield novel channels for energy transfer. These results pave the way to the design of innovative structures to manage the noncontact heat exchanges.

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I. INTRODUCTION

In the last decade, a considerable amount of work has been dedicated to the study of metamaterials and left-handed materials.[1] These composite materials are constituted by arrays of artificial electric and magnetic resonators.[2] When the photon wavelength which propagate in these materials is large enough compared to the size of their representative unit cell, they behave like an effective medium in which both permittivity and permeability exhibit resonances. In some particular conditions many metamaterials display previously unrealizable optical properties such as negative refraction[3] or the reversed Doppler effect.[4] These properties have been recently exploited to break old paradigms in physics. For instance, some magnetodielectric materials have been designed to conceive superlenses[5] which have a spatial resolution below the half wavelength or to create coating which makes an object invisible[6,7] to an external observer over a given spectral range.

Metamaterials have necessary dielectric and magnetic losses and radiate an electromagnetic field in their surrounding when heated. A still open question is how energy is exchanged between two metamaterials in nonequilibrium situation and what are the physical mechanisms involved in this transfer. Radiative and nonradiative heat transfers between noncontact purely dielectric hot bodies at different temperature have been intensively studied in the past.[8] When the separation distance between two bodies is of the order of their Wien's wavelength, near-field (NF) effects enhance heat exchanges due to tunneling of evanescent waves.[9–16] When the bodies support surface modes such as surface polaritons, transfer is strongly enhanced far beyond the prediction of the Planck's theory of blackbody and heat transfer is mostly monochromatic.[17] On the other side, interaction of magnetic dipoles between metallic materials lead to a frequency broad transfer mainly though *s*-polarized waves.[9,18] Near-field thermal radiation and heat transfer have extended beyond theoreticians and have been detected recently through different experimental devices.[19–22] Applications about near-field heat transfer are promising and could lead to the design of new energy sources such as thermophotovoltaic systems.[23,24] In this context, one can wonder if metamaterials are a promising way to enhance noncontact heat transfer.

In this paper we explore by means of the fluctuational electrodynamics radiative heat transfer between two interacting metamaterials which support both magnetic and dielectric multipoles of any order. We first recall how, from the Green dyadic relating an electric current density to the electric field, we can define three other Green dyadics relating any current (electric or magnetic) to any field (electric or magnetic). Without loss of generality, we examine radiative heat transfer between two plane semi-infinite bodies. With the help of Green dyadics expressions and fluctuation-dissipation theorem (FDT) for electric and magnetic currents density, we derive the radiative heat transfer general expression between any plane semi-infinite bodies. We finally discuss the peculiarities of heat transfer in magnetodielectric materials. We show, in particular, that new channels are opened to transfer heat in the near field due to the simultaneous presence of dielectric and magnetic resonance.

II. ELECTROMAGNETIC FIELD GREEN DYADIC

Let us consider an homogeneous metamaterial define by its permittivity $\epsilon(\omega)=\epsilon_0\epsilon_r(\omega)$ and permeability $\mu(\omega)=\mu_0\mu_r(\omega)$. In the presence of electric and magnetic currents $\mathbf{j}_e$ and $\mathbf{j}_m$, propagation equation for a monochromatic electric field $\mathbf{E}$ reads

$$\nabla\times(\nabla\times\mathbf{E})+\frac{\omega^2}{c^2}\mu_r\epsilon_r\mathbf{E}=i\mu\omega\mathbf{j}_e-\nabla\times\mathbf{j}_m. \qquad (1)$$

Introducing $\overleftrightarrow{\mathbf{G}}_{Ee}(\mathbf{r},\mathbf{r}')$ the Green dyadic relating elementary electric currents in $\mathbf{r}'$ to the electric field they create in $\mathbf{r}$, we have

$$\mathbf{E}(\mathbf{r},\omega)=\int d^3\mathbf{r}'i\omega\mu(\mathbf{r}')\overleftrightarrow{\mathbf{G}}_{Ee}(\mathbf{r},\mathbf{r}',\omega)\mathbf{j}_e(\mathbf{r}')d^3\mathbf{r}'. \qquad (2)$$

$\overleftrightarrow{\mathbf{G}}_{He}(\mathbf{r},\mathbf{r}')$ $[\mathbf{H}(\mathbf{r},\omega)=\int d^3\mathbf{r}'\overleftrightarrow{\mathbf{G}}_{He}(\mathbf{r},\mathbf{r}',\omega)\mathbf{j}_e(\mathbf{r}')d^3\mathbf{r}']$ can easily be obtained from $\overleftrightarrow{\mathbf{G}}_{Ee}(\mathbf{r},\mathbf{r}')$. Indeed, if we only take into account the response to electric currents, monochromatic fields are related through $\mathbf{H}=\nabla\times\mathbf{E}/(i\mu\omega)$. Thus,

$$\overleftrightarrow{\mathbf{G}}_{He}(\mathbf{r},\mathbf{r}',\omega)=\frac{\mu(\mathbf{r}')}{\mu(\mathbf{r})}\nabla_{\mathbf{r}}\times\overleftrightarrow{\mathbf{G}}_{Ee}(\mathbf{r},\mathbf{r}',\omega). \tag{3}$$

Let us now treat the case of an electric field created by magnetic currents. Recalling propagation Eq. (1), it is possible to calculate the electric field created by magnetic currents from $\overleftrightarrow{\mathbf{G}}_{Ee}$ replacing $\mathbf{j}_e(\mathbf{r}')$ by $-\nabla\times\mathbf{j}_m(\mathbf{r}')/[i\mu(\mathbf{r}')\omega]$. We find

$$\mathbf{E}(\mathbf{r},\omega)=\int d^3\mathbf{r}'\overleftrightarrow{\mathbf{G}}_{Ee}(\mathbf{r},\mathbf{r}',\omega)[-\nabla_{\mathbf{r}'}\times\mathbf{j}_m(\mathbf{r}')]d^3\mathbf{r}'. \tag{4}$$

Performing an integration by parts on this last expression, one shows after little algebra that

$$\overleftrightarrow{\mathbf{G}}_{Em}(\mathbf{r},\mathbf{r}'\omega)=[\nabla_{\mathbf{r}'}\times{}^T\overleftrightarrow{\mathbf{G}}_{Ee}(\mathbf{r},\mathbf{r}',\omega)]^T, \tag{5}$$

where ^{T} denote the transpose operator. Taking the curl of the electric field created from a magnetic field $\{\mathbf{H}(\mathbf{r},\omega)=\nabla_{\mathbf{r}}\times\mathbf{E}(\mathbf{r},\omega)/[i\mu(\mathbf{r})\omega]\}$, one obtains the magnetic field created from a magnetic current. As $\mathbf{H}(\mathbf{r},\omega)=\int d^3\mathbf{r}'i\omega\epsilon(\mathbf{r}')\overleftrightarrow{\mathbf{G}}_{Hm}(\mathbf{r},\mathbf{r}',\omega)\mathbf{j}_m(\mathbf{r}')$,

$$\overleftrightarrow{\mathbf{G}}_{Hm}(\mathbf{r},\mathbf{r}',\omega)=\frac{\nabla_{\mathbf{r}}}{\epsilon(\mathbf{r}')\mu(\mathbf{r})\omega^2}\times[\nabla_{\mathbf{r}'}\times{}^T\overleftrightarrow{\mathbf{G}}_{Ee}(\mathbf{r},\mathbf{r}',\omega)]^T. \tag{6}$$

Relations (3), (5), and (6) between Green dyadics are general, valid for an arbitrary geometry and only assume that the materials optical response is local.

III. RADIATIVE HEAT TRANSFER BETWEEN TWO MAGNETOELECTRIC SEMI-INFINITE BODIES

A. Theory

Let us now consider the situation of a semi-infinite flat body (medium 1) orthogonal to z axis separated from another semi-infinite flat body (medium 2) by a vacuum gap (medium 3) of thickness d. We propose to calculate heat radiatively transferred from medium 1 in medium 2. In this geometry, Green dyadic $\overleftrightarrow{\mathbf{G}}_{Ee}(\mathbf{r},\mathbf{r}',\omega)$, written by various authors[9,25,26] and all other dyadics can be obtained with expressions developed above. Indeed the Green dyadic $\overleftrightarrow{\mathbf{G}}^{Ee}(\mathbf{r},\mathbf{r}',\omega)$ reads

$$\overleftrightarrow{\mathbf{G}}^{Ee}(\mathbf{r},\mathbf{r}',\omega)=\frac{i}{8\pi^2}\int\frac{d^2\mathbf{K}}{\gamma_2}(\hat{s}t_{21}^s\hat{s}+\hat{p}_1^+t_{21}^p\hat{p}_2^+)e^{i\mathbf{K}.(\mathbf{R}-\mathbf{R}')}e^{i\gamma_1(z-d)}e^{-i\gamma_2 z'}. \tag{7}$$

In the previous expression, wave-vector components are $\mathbf{k}_i=(\mathbf{K},\gamma_i)$ with $K^2+\gamma_i^2=\epsilon_i\mu_ik_0^2$. We also have $\hat{s}=\hat{K}\times\hat{z}$ and $\hat{p}_i^\pm=(K\hat{z}\mp\gamma_i\hat{K})/(n_ik_0)$, where $\hat{K}=\mathbf{K}/K$ and $\hat{z}$ is the unit vector along direction z. Relations developed in Sec. II gives all Green dyadics expression for the present geometry

$$\overleftrightarrow{\mathbf{G}}^{He}(\mathbf{r},\mathbf{r}',\omega)=-\frac{n_1\mu_2\omega}{8\pi^2\mu_1c}\int\frac{d^2\mathbf{K}}{\gamma_2}\times(-\hat{p}_1^+t_{21}^s\hat{s}+\hat{s}t_{21}^p\hat{p}_2^+)e^{i\mathbf{K}.(\mathbf{R}-\mathbf{R}')}e^{i\gamma_1(z-d)}e^{-i\gamma_2 z'}, \tag{8}$$

$$\overleftrightarrow{\mathbf{G}}^{Em}(\mathbf{r},\mathbf{r}',\omega)=\frac{n_2\omega}{8\pi^2c}\int\frac{d^2\mathbf{K}}{\gamma_2}\times(\hat{s}t_{21}^s\hat{p}_2^+-\hat{p}_1^+t_{21}^p\hat{s})e^{i\mathbf{K}.(\mathbf{R}-\mathbf{R}')}e^{i\gamma_1(z-d)}e^{-i\gamma_2 z'}, \tag{9}$$

$$\overleftrightarrow{\mathbf{G}}^{Hm}(\mathbf{r},\mathbf{r}',\omega)=\frac{in_1n_2}{8\pi^2c^2\epsilon_2\mu_1}\int\frac{d^2\mathbf{K}}{\gamma_2}\times(\hat{s}t_{21}^p\hat{s}+\hat{p}_1^+t_{21}^s\hat{p}_2^+)e^{i\mathbf{K}.(\mathbf{R}-\mathbf{R}')}e^{i\gamma_1(z-d)}e^{-i\gamma_2 z'}. \tag{10}$$

In this geometry, the field has the form

$$\mathbf{A}=C\int d^3\mathbf{r}'\int d^2\mathbf{K}\overleftrightarrow{\mathbf{A}}e^{i\mathbf{K}.(\mathbf{R}-\mathbf{R}')}e^{i\gamma_1(z-d)}e^{-i\gamma_2 z'}\mathbf{j}(\mathbf{r}'). \tag{11}$$

Any quadratic combination of the fields will have the following form:

$$A_iB_j^*=D\int d^3\mathbf{r}'d^3\mathbf{r}''\int d^2\mathbf{K}d^2\mathbf{K}'A_{ik}j_k(\mathbf{r}')B_{jl}^*j_l^*(\mathbf{r}'')\times e^{i\mathbf{K}.(\mathbf{R}-\mathbf{R}')}e^{i\gamma_1(z-d)}e^{-i\gamma_2 z'}e^{-i\mathbf{K}'.(\mathbf{R}-\mathbf{R}'')}e^{i\gamma_1'(z-d)}e^{-i\gamma_2' z'}. \tag{12}$$

When one calculates the heat flux between material 2 and material 1, one has to calculate Poynting vector in medium 1, which originates from medium 2. In the chosen geometry, heat flux is equal to the z component of the Poynting vector,

$$\langle S_z\rangle=2\,\mathrm{Re}(E_xH_y^*-E_yH_x^*). \tag{13}$$

Note that this expression is four times the classical expression due to the definition of the Fourier transform and to the fact that for real signals, only positive frequencies contribute to the heat transfer. Note also in Eq. (13) that an ensemble average is taken on all the system realizations. When this operation is carried out, FDT for currents is applied. In our problem, current fluctuations are due to thermal fluctuations in the system. Indeed, fluctuations in the material initiate charge movements that originate thermal electric dipoles and currents. Moreover, in the case of magnetic materials, these fluctuations initiate also fluctuating magnetic dipoles which temporal derivatives are fluctuating magnetic currents. It can be shown that fluctuating currents correlation functions depends on the imaginary part of the dielectric function for electric currents and of the imaginary part of the permeability for the magnetic currents.[27,28] Magnetic and electric currents are not correlated. The following correlation functions are taken following FDT,[28]

$$\langle j_i^e(\mathbf{r})j_j^{e*}(\mathbf{r}')\rangle=\frac{\mathrm{Im}(\epsilon)\omega\Theta(\omega,T)}{\pi}\delta_{ij}\delta(\mathbf{r}-\mathbf{r}'), \tag{14}$$

$$\langle j_i^m(\mathbf{r})j_j^{m*}(\mathbf{r}')\rangle=\frac{\mathrm{Im}(\mu)\omega\Theta(\omega,T)}{\pi}\delta_{ij}\delta(\mathbf{r}-\mathbf{r}'), \tag{15}$$

$$\langle j_i^e(\mathbf{r}) j_j^{m*}(\mathbf{r}')\rangle = 0, \tag{16}$$

where $\Theta(\omega,T)=\hbar\omega/[\exp(\hbar\omega/k_bT)-1]$ is the mean energy of an oscillator. A consequence of the absence of correlations between magnetic and electric currents is that radiative heat transfer will only contains from one hand products of elements of $\overleftrightarrow{\mathbf{G}}^{Ee}$ and $\overleftrightarrow{\mathbf{G}}^{He}$ and from the other hand products of elements of $\overleftrightarrow{\mathbf{G}}^{Em}$ and $\overleftrightarrow{\mathbf{G}}^{Hm}$. After integration over $d^3\mathbf{r}''$, d^3r', and $d^2\mathbf{K}'$ and some simplifications, the heat flux now reads

$$\langle S_z\rangle = \frac{\Theta}{8\pi^3}\int \frac{d^2K}{|\gamma_2|^2}\left[\frac{|t_{21}^s|^2\mathrm{Re}(\mu_1\gamma_1^*)\mathrm{Re}(\mu_2\gamma_2^*)}{|\mu_1|^2} + \frac{|t_{21}^p|^2\mathrm{Re}(\epsilon_1\gamma_1^*)\mathrm{Re}(\epsilon_2\gamma_2^*)|\mu_2|^2c^4}{|n_1|^2|n_2|^2}\right]. \tag{17}$$

There exist relations between Fresnel coefficients. First of all, transmission coefficient from medium 2 to medium 1 reads

$$t_{21} = \frac{t_{23}t_{32}e^{i\gamma_3 d}}{1 - r_{31}r_{32}e^{2i\gamma_3 d}}. \tag{18}$$

Moreover

$$\mathrm{Re}(\mu_3^*\gamma_3)(1-|r_{31}^s|^2) + 2\,\mathrm{Im}(\mu_3^*\gamma_3)\mathrm{Im}(r_{31}^s) = \mathrm{Re}(\mu_1^*\gamma_1) \times |t_{31}^s|^2\frac{|\mu_3|^2}{|\mu_1|^2}, \tag{19}$$

$$\mathrm{Re}(\mu_3^*\gamma_3)(1-|r_{32}^s|^2) + 2\,\mathrm{Im}(\mu_3^*\gamma_3)\mathrm{Im}(r_{32}^s) = \mathrm{Re}(\mu_2^*\gamma_2) \times |t_{23}^s|^2\frac{|\gamma_3|^2}{|\gamma_2|^2}, \tag{20}$$

$$\mathrm{Re}(\epsilon_3^*\gamma_3)(1-|r_{31}^p|^2) + 2\,\mathrm{Im}(\epsilon_3^*\gamma_3)\mathrm{Im}(r_{31}^p) = \mathrm{Re}(\epsilon_1^*\gamma_1) \times |t_{31}^p|^2\frac{|n_3|^2}{|n_1|^2}, \tag{21}$$

$$\mathrm{Re}(\epsilon_3^*\gamma_3)(1-|r_{32}^p|^2) + 2\,\mathrm{Im}(\epsilon_3^*\gamma_3)\mathrm{Im}(r_{32}^p) = \mathrm{Re}(\epsilon_2^*\gamma_2) \times |t_{23}^p|^2\frac{|n_2|^2}{|n_3|^2}\frac{|\epsilon_3|^2}{|\epsilon_2|^2}\frac{|\gamma_3|^2}{|\gamma_2|^2}. \tag{22}$$

Inside the cavity $\epsilon_3=\epsilon_0$ and $\mu_3=\mu_0$. If $K<\omega/c$, $\mathrm{Re}(\gamma_3)=\gamma_3$ and $\mathrm{Im}(\gamma_3)=0$. On the contrary, if $K>\omega/c$, $\mathrm{Im}(\gamma_3)=|\gamma_3|$ and $\mathrm{Re}(\gamma_3)=0$. One can separate propagative and evanescent waves contributions and s and p contributions. This gives the following heat flux expression of heat emitted in medium 1 of temperature T_1 and dissipated in medium 2. For s and p polarization, the propagative contributions reads

$$\langle S_z^{s,p}\rangle = \frac{\Theta(\omega,T_1)}{8\pi^3}\int_0^{\omega/c} 2\pi K dK \frac{(1-|r_{31}^{s,p}|^2)(1-|r_{32}^{s,p}|^2)}{|1-r_{31}^{s,p}r_{32}^{s,p}e^{2i\gamma_3 d}|^2}. \tag{23}$$

Evanescent contribution reads

$$\langle S_z^{s,p}\rangle = \frac{\Theta(\omega,T_1)}{\pi^2}\int_{\omega/c}^{\infty} K dK \frac{\mathrm{Im}(r_{31}^{s,p})\mathrm{Im}(r_{32}^{s,p})}{|1-r_{31}^{s,p}r_{32}^{s,p}e^{2i\gamma_3 d}|^2}e^{-2|\gamma_3|d}. \tag{24}$$

These expressions are the same as those established between two nonmagnetic materials.[9,26] Note that this kind of result has already been derived in the past in a different way.[10,29] Finally, heat flux exchanged is obtained replacing Θ by $\Theta(\omega,T_1)-\Theta(\omega,T_2)$ in Eqs. (23) and (24).

B. Results

We now proceed to the evaluation of the near-field radiative heat transfer between two semi-infinite materials. As outlined in Sec. I, two nonmagnetic semi-infinite materials separated by a subwavelength distance[9] exhibit heat transfer enhancement. For some materials such as metals, near-field transfer is dominated by modes contributing essentially to the magnetic density of energy.[30] On their side, polar materials such as SiC or silica exhibit surface waves and a strong density of states at wavelengths of the order of tens of microns. When two of such semi-infinite bodies are put at subwavelength distances, energy transfer occurs, essentially at the polaritons frequencies so that heat transfer is almost monochromatic. Note that this "polaritonlike transfer" only occurs in p polarization for nonmagnetic materials.[31] In the case of a magnetodielectric material, surface waves exist in both p and s polarization. Their dispersion relation read

$$K_p = \frac{\omega}{c}\sqrt{\frac{\epsilon_r(\epsilon_r-\mu_r)}{\epsilon_r^2-1}}, \quad K_s = \frac{\omega}{c}\sqrt{\frac{\mu_r(\mu_r-\epsilon_r)}{\mu_r^2-1}}. \tag{25}$$

These relations are obtained if we look for the conditions for which there is a wave in the absence of excitation, i.e., if the interfaces reflection coefficient are infinite. This corresponds to a material reflection coefficient pole.[26] Close to materials, the electromagnetic density of states, related to the imaginary part of reflection coefficients[12] increases in the presence of surface waves and enhances evanescent heat transfer, Eq. (24).

As effective reduced permittivity and permeability, we use the following expressions derived for an artificial array of wires and split rings[2]

$$\epsilon_r = 1 - \frac{\omega_p^2}{\omega(\omega+i\gamma_e)}, \quad \mu_r = 1 - \frac{F\omega^2}{\omega^2-\omega_0^2+i\gamma_m\omega}, \tag{26}$$

where F is the split rings filling factor. In these expression ω_p is an equivalent plasma pulsation. Note that it is different from the plasma pulsation of a metal. Here, $\omega_p=ne^3/(\epsilon_0 m)$, where n is the number of free electrons per unit volume. In a metamaterial made of an array of wire and split rings, this equivalent plasma frequency can be shifted in the infrared range if the wires are sufficiently diluted in the air, i.e., if the metal volume fraction of the metal constituting the metamaterial is sufficiently low. On the other way, the technological limit of the existence of magnetic resonance in a wavelength domain is actually the scale at which it is possible to make wire rings which has to be small compared to the wavelength considered. Recent studies have shown that both resonances,

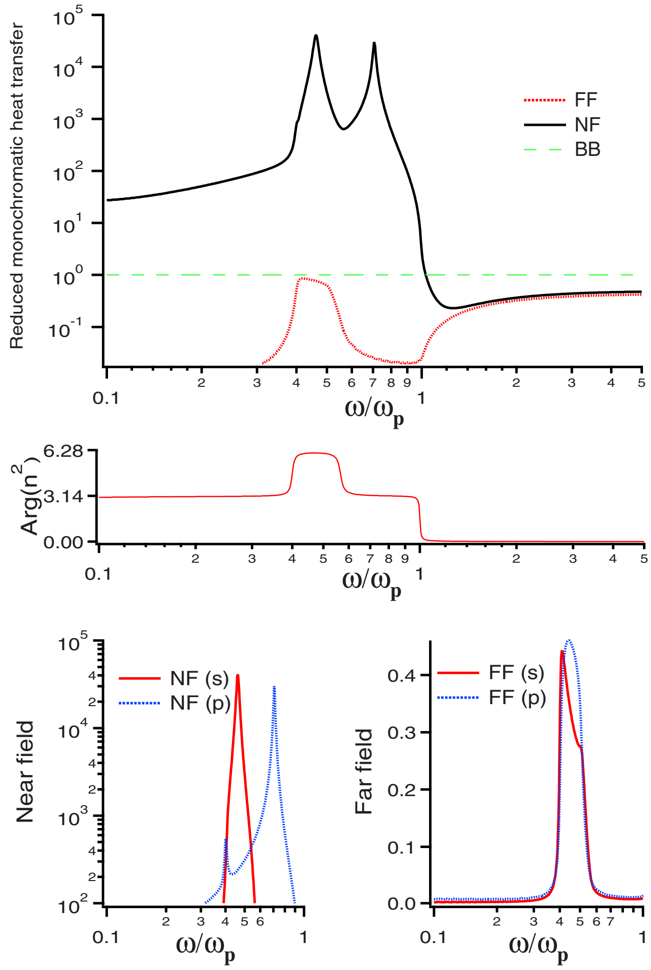

FIG. 1. (Color online) Upper figure: Reduced monochromatic radiative heat transfer (normalized to heat transfer between two perfect BB) between two identical metamaterial with the following parameters: $\omega_0=0.4\omega_p$ and $\gamma_e=\gamma_m=0.01\omega_p$. Materials are maintained at two different temperatures (0.406 and 0.380 $\hbar\omega_p/k_b$). $d=53.1\lambda_p$ in the FF. $d=5.31\times10^{-3}\lambda_p$ in the NF. Central figure: $\arg(\epsilon_r\mu_r)$ versus angular frequency. Left lower figure: p- and s-polarization contribution to the near-field heat transfer. Right lower figure: p- and s-polarization contribution to the far-field heat transfer.

electric and magnetic could be in the infrared range.[32,33] In order to present results that do not depend on the nowadays technology limitations, we made calculations where distances are related to the effective plasma wavelength and frequencies are related to the effective plasma pulsation. We will first take the following parameters $\omega_0=0.4\omega_p$, $\gamma_e=\gamma_m=0.01\omega_p$, and $F=0.5$ and calculate heat transfer between two identical metamaterials heated at 0.406 and 0.380 $\hbar\omega_p/k_b$, respectively. Introducing $\lambda_p=2\pi c/\omega_p$ the plasma wavelength, we take $d=5.31\times10^{-3}\lambda_p$ for NF calculation and $d=53.1\lambda_p$ for far-field (FF) calculations. Note that these materials are left handed and have a negative index for $\omega_0<\omega<0.6\omega_p$. Following Skaar,[34] an absorbing material is left handed if and only if $\arg(n^2)=\arg(\epsilon_r)+\arg(\mu_r)>\pi$ and $n=\sqrt{|\epsilon_r||\mu_r|}\exp\{i[\arg(\epsilon_r)+\arg(\mu_r)]/2\}$. Three different behaviors can be recognized in Fig. 1 central figure: $\arg(\epsilon_r\mu_r)$

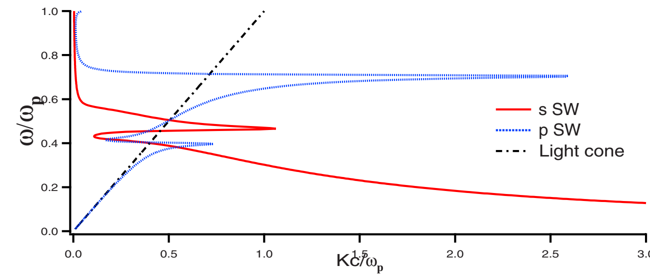

FIG. 2. (Color online) Surface waves dispersion relations for s polarization (s SW) and p polarization (p SW) obtained from Eq. (25) in which Re(K) is plotted. Dashed-dotted line represents the light cone.

>0 and close to zero corresponding to classical absorbing refracting materials, $\arg(\epsilon_r\mu_r)$ close to π corresponding to highly reflective materials (metallic behavior) and $\arg(\epsilon_r\mu_r)$ slightly lower than 2π corresponding to absorbing negative index materials. Above this latter curve are represented the reduced radiative heat transfer which is the ratio of radiative heat transfer between two identical materials to the heat transfer between two blackbodies (BB). In far field, we note that heat transfer is much smaller than heat transfer between blackbodies at frequencies where materials are highly reflective.

At frequencies where the material is absorbing, with a negative index or not, we note that heat transfer approaches blackbody heat transfer due to the fact that materials poorly reflect radiation and emit it efficiently. In near field, we note that radiative heat transfer is enhanced by several orders of magnitude compare to heat transfer in far field. Heat transfer mainly occurs at two particular frequencies corresponding to frequencies where ϵ_r and μ_r approach -1. A careful look at p-polarized evanescent waves contribution shows the presence of a secondary peak (lower left figure of Fig. 1).

Figure 2 shows s- and p-polarization surface wave dispersion relations) obtained from Eq. (25) in which Re(K) is plotted. We can identify the three peaks corresponding to the three zones where dispersion relations are below the light line with a flat asymptote. These asymptotes appear when we put optical properties in dispersion relation (26). When we neglect losses in the materials, dispersion relation reads for p polarization,

$$K=\frac{\omega}{c}\sqrt{\frac{[1-(\omega_p/\omega)^2][F\omega^4-\omega_p^2(\omega^2-\omega_0^2)]}{\omega_p^2[\omega^2-\omega_0^2][(\omega_p/\omega)^2-2]}}, \qquad (27)$$

whereas it reads in s polarization,

$$K=\frac{\omega}{c}\sqrt{\frac{[\omega^2(1-F)-\omega_0^2][\omega^2(\omega_p^2-F\omega^2)-\omega_p^2\omega_0^2]}{F\omega^2[\omega^2(F-2)+2\omega_0^2]}}. \qquad (28)$$

Two asymptotes occur in p polarization for $\omega=\omega_p/\sqrt{2}$, where ϵ_r approaches -1 and for $\omega=\omega_0$ for which the permeability μ_r is very large. In s polarization, an asymptote appears for $\omega=\sqrt{2}\omega_0/\sqrt{2-F}$ for which μ_r approaches -1.

The peak corresponding to $\epsilon_r=-1$ is the classical surface polariton peak. Heat is transferred through the coupling of

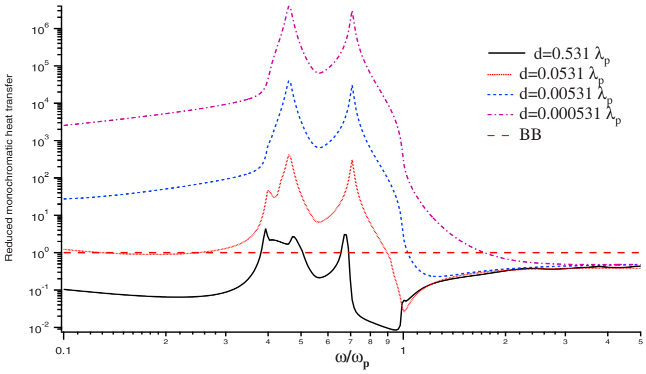

FIG. 3. (Color online) Reduced monochromatic heat transfer between metamaterials with the following parameters: $\omega_0=0.4\omega_p$ and $\gamma_e=\gamma_m=0.01\omega_p$. Materials are maintained at two different temperatures (0.406 and 0.380 $\hbar\omega_p/k_b$). The transfer is calculated for various separation distance. $0.531\lambda_p$, $5.31\times10^{-2}\lambda_p$, $5.31\times10^{-3}\lambda_p$, and $5.31\times10^{-4}\lambda_p$.

surface waves at subwavelength distances. Note that this peak in the near field corresponds to a nonemitting zone in the far field where ϵ_r is negative and μ_r close to one. In this case, optical index is close to a pure imaginary number and the material is highly reflective. The peak which appears at the frequency where $\mu_r=-1$ is due to the magnetic equivalent of the surface polariton resonance. Here, strong near-field emission at the resonance frequency corresponds to a strong emission in the far field. In the present case, the medium is not highly reflective but left handed with a rather strong emission. Change in refraction laws for left-handed material does not affect thermal emission. Both resonances can be denominated "polaritonlike" waves. They correspond to a dipolar electric or magnetic excitation propagating along the interface. Contrary to the preceding resonances, the resonance responsible for the secondary peak only exists if the materials are both dielectric and magnetic (ϵ_r and μ_r different from 1). This corresponds to a frequency where μ_r is very large so that magnetic excitation at this frequency gives a strong magnetization as if magnetic dipoles were aligned

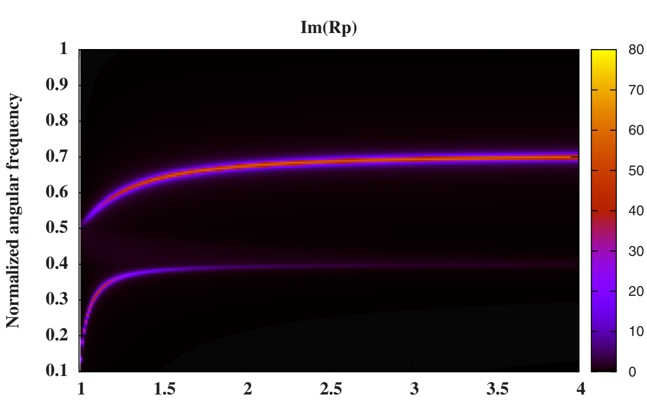

FIG. 4. (Color online) Imaginary part of the reflection coefficient of a semi-infinite metamaterial with the parameters of Fig. 1. $\mathrm{Im}(r_p)$ is represented in the plane (K/k_0 and ω/ω_p). Highest asymptote corresponds to the polariton resonance whereas the lowest correspond to the frequency where μ is large.

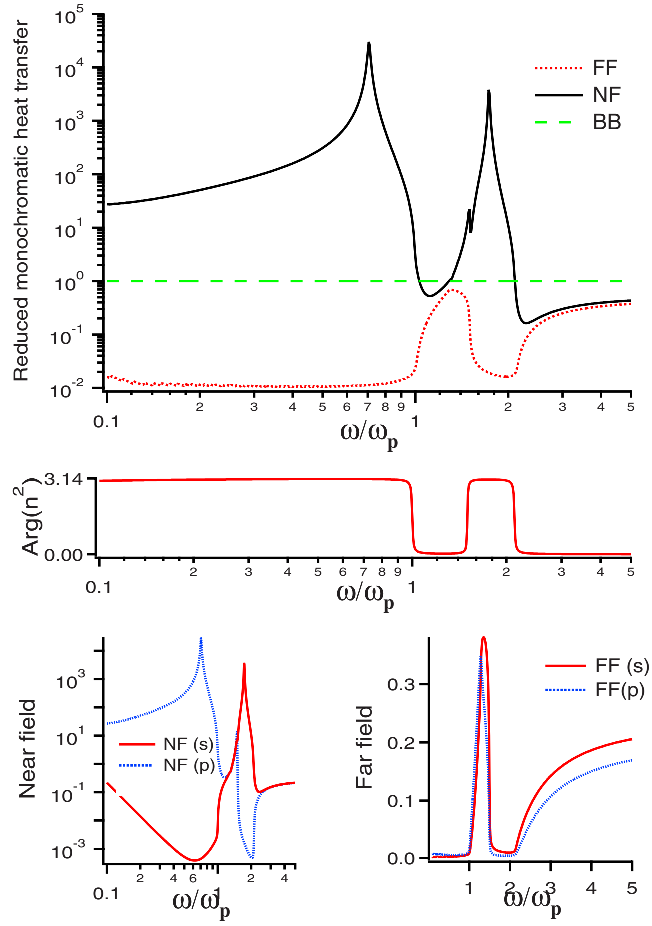

FIG. 5. (Color online) Upper figure: Reduced monochromatic radiative heat transfer between two identical metamaterials with $\omega_0=1.5\omega_p$ and all other parameters identical to Fig. 1. Central figure: $\arg(\epsilon_r\mu_r)$ versus ω/ω_p. Left lower figure: *p*- and *s*-polarization contribution to the near-field heat transfer. Right lower figure: *p*- and *s*-polarization contribution to the far-field heat transfer.

like in a ferromagnetic material. As polaritonlike waves, these last resonant waves are longitudinal waves. It can be seen that the respective influence of these two kinds of waves depends on the sample separation distance.

In Fig. 3 are plotted the reduced monochromatic heat transfer for $0.531\lambda_p$, $5.31\times10^{-2}\lambda_p$, $5.31\times10^{-3}\lambda_p$, and $5.31\times10^{-4}\lambda_p$. For the highest separation distance, we note three peaks of comparable intensity. The resonance due to the large permittivity (left peak) is as high as the one due to polaritons (the two right peaks). When the separation distance reduces, the polariton peaks increases as $1/d^2$ whereas the peak due to the large μ increases slower and is finally overtaken by the other contributions. Analysis of asymptotic expressions of the interface reflection coefficients allows to understand this behavior. Indeed, for large K, $r_s\approx r_s^{asymp}=(\mu-1)/(\mu+1)$ and $r_p\approx r_p^{asym}=(\epsilon-1)/(\epsilon+1)$. At small distances, the integral in expression (24) is dominated by the contribution of large K. As $\gamma_3\approx iK$ for large K and introducing $u=Kd$,

$$\int_{\omega/c}^{\infty} KdK \frac{\mathrm{Im}(r_{31}^{s,p})\mathrm{Im}(r_{32}^{s,p})}{|1 - r_{31}^{s,p} r_{32}^{s,p} e^{2i\gamma_3 d}|^2} e^{-2|\gamma_3|d} \approx \int_{\omega/c}^{\infty} KdK \frac{\mathrm{Im}(r_{s,p}^{asymp})\mathrm{Im}(r_{s,p}^{asymp})}{|1 - r_{s,p}^{asymp} r_{s,p}^{asymp} e^{-2Kd}|^2} e^{-2Kd} \approx \frac{1}{d^2}\int_0^{\infty} udu \frac{\mathrm{Im}(r_{s,p}^{asymp})\mathrm{Im}(r_{s,p}^{asymp})}{|1 - r_{s,p}^{asymp} r_{s,p}^{asymp} e^{-2u}|^2} e^{-2u}. \quad (29)$$

The last integral is a number that only depends on the values of the asymptotic reflexions coefficient in the cavity at a certain angular frequency ω. Of course, the reflexion coefficients will remain high when $\mu \approx -1$ or $\epsilon \approx -1$. There will thus be strong peaks at this frequency with a $1/d^2$ dependency. This is not the case when μ is very large. An analysis of $\mathrm{Im}(r_p)$ in the K, ω plane as depicted in Fig. 4 shows that this quantity is large at the frequency where μ_r is large but for a limited interval in K of a few k_0. In contrast $\mathrm{Im}(r_p)$ remains large for high values of K at the polariton frequency.

At large distances, contribution of the "large μ" mode is comparable to the one of polaritons. At smaller distances, the contribution is dominated by polaritons since they are the only modes that have a strong contributions for large K. We can conclude that these different modes have not the same domain of existence. Polariton like waves are dominant in the extreme near-field regime whereas the large μ resonance is dominant in the near field.

Let us now study radiative heat transfer between magnetodielectric metamaterials with no negative index frequency domain. We take the same parameters than in the preceding case excepted that we shift the magnetic resonance toward $\omega_0 = 1.5\omega_p$. We note in Fig. 5 that emission in the far field is weak when the material is highly reflective [i.e., when $\arg(\epsilon\mu)$ is close to π] and high when $\arg(\epsilon\mu)$ is close to zero.

In near field, similarly to the previous case, radiative heat transfer exhibits three peaks. Two of them correspond to frequencies where ϵ_r or μ_r are equal to -1 and the third tiny peak corresponds to a frequency where μ_r becomes very large. Here, the two largest emission peaks in the near field correspond to frequency domain in the far field where materials are reflective. We therefore see that it does not seem to have a different behavior between negative index metamaterial and positive index ones. Radiative heat transfer is determined by the medium reflection in the far field and by the presence of surface waves in the near field. However, we have seen that magnetic response opens new frequency channels for near-field heat transfer whatever is the sign of the optical index. Magnetodielectric materials therefore have a potential to increase noncontact heat transfer and to be used in any device which has the goal to enhance this kind of transfer. Moreover, metamaterials can be built in order to obtain the transfer at given frequencies. This could pave the way to the design of thermophotovoltaic devices in which heat transferred at one frequency could be transformed with high efficiency.

IV. CONCLUSION

In this paper, we have studied radiative heat transfer between two semi-infinite magnetodielectric materials by means of fluctuational electrodynamics. From the radiative heat flux expression, we have shown that radiative heat transfer is enhanced in the near field when materials exhibit surface waves. We have seen that surface wave resonances occur when ϵ_r or μ_r are equal to -1 or when μ_r becomes very large. In the first two conditions (ϵ_r or μ_r close to -1), we are in the presence of electric or magnetic polaritons which contribution to the heat transfer increases as the square of the inverse of the separation distance. The large μ_r case is different and only occurs in the case of magnetodielectric materials, i.e., for ϵ_r and μ_r simultaneously different from 1. The effect of this last resonance increases less than the polariton ones and becomes negligible at ultrashort distances. The fact that the material exhibits a negative index does not play any particular role. This is the presence of dielectric or magnetic resonance which is essential. Radiative heat transfer in the near field is dominated by these resonances contributions so that new frequency channels are open for magnetodielectric materials.

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